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## Looking for stability

### Advances on spectrum-based stability and stabilization for uncertain time-delay systems

#### Introduction / Objective

Time-delay systems model widespread phenomena, ranging from engineering to life sciences applications, where future events are not only determined by the present but also by the past. The stability properties of these systems determine whether every solution will remain close to an equilibrium or be repelled away; and the stability optimization drives every solution to decay to an equilibrium as fast as possible. Therefore, stability and stabilization are fundamental methods for the construction of reliable machines, or to quantify parameters in mathematical models. The objective of my PhD is to develop reliable numerical methods for the stability assessment and stability optimization of time-delay system.

#### Research Methodology

We describe all the solutions of a time-delay system by operators so that the stability conditions are determined by intrinsic quantities, the eigenvalues, of these operators. Indeed, the asymptotic growth or decay rate of the solution towards an equilibrium depends on these eigenvalues. Hence, the system stability can be inferred from the location of the infinite-dimensional operator eigenvalues, and their relocation might provide stronger stability properties (stabilization).

To accurately compute the stability assessment, the eigenvalue computation is handled by a two-phase method. The first phase discretizes the infinite-dimensional operator into a standard eigenvalue problem by a collocation method. Then, in the second phase, the estimated eigenvalues are refined up to machine precision by local root finding methods.

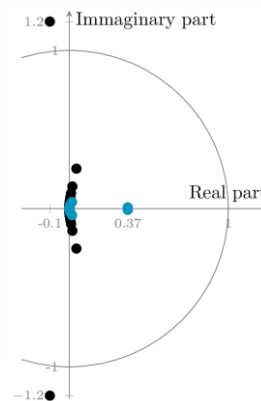
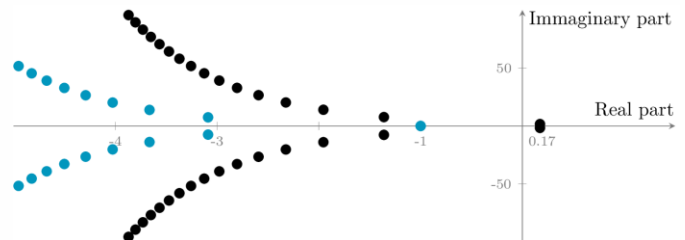
The stabilization relocates in the complex plane the operator eigenvalues, improving the asymptotic convergence rate of the solutions to the equilibrium. This relocation is handled by an optimization software HANSO (Hybrid Algorithm for Non-Smooth optimization) which requires the objective function and its gradient, whenever it exists.

#### Results & Conclusions

1. We develop a novel stabilization method for time-delay systems whose parameters are affected by uncertainties, modeled by a random vector. Compared to standard stabilization methods, our method shows better robustness properties, which are not conservative compared to worst-case approaches. Moreover, it can provide different system dynamics.

2. We develop a novel stability assessment and stabilization method for periodic time-delay systems, whose delays and period are commensurable.

The methods has been successfully tested on several examples and engineering applications. Indeed, we improve the stability properties of a heat-transfer set up, of a mechanical system with a delayed resonator absorbing external harmonic oscillations, and of a large-scale milling model describing the periodic interaction between a rotating cutter and a visco-elastic workpiece.



The blue (black) dots represent the eigenvalues for a stable (unstable) linear time-delay system. In the figure above, the time-invariant system is asymptotically stable if and only if all the infinitesimal generator eigenvalues are contained in the left-complex plane. In the figure on the left, the time-periodic system is asymptotically stable if and only if the monodromy operator nonzero eigenvalues are strictly contained in the complex unit circle.

#### Major publications

L. Fenzi & W. Michiels. *Robust stability optimization for delay systems in a probabilistic framework*. Linear Algebra and its Applications, 526, 1-26 (2017).

W. Michiels & L. Fenzi. *Spectrum-based stability analysis and stabilization of a class of time-periodic time delay systems* (submitted)